MATH EDUCATORS SHARE IDEAS THAT CHANGED THEIR CLASSROOMS

MY BEST IDEA

MATH • VOLUME 1

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Rubicon

TARGET AUDIENCE:
School administrators, K–12 teachers

MY BEST IDEA

RUTH BEATTY AND CHRISTINA RUDDY

MATHEMATICS EXPLORATION THROUGH ALGONQUIN BEADWORK

How can we use the mathematics inherent in Indigenous cultural practices to enhance mathematical content knowledge? Follow the experiences of a group of educators and Community members as, using beadwork, they guide students not only in learning about foundational concepts such as unitizing but also words and phrases in Algonquin related to the activity of beading. In the process, everyone gains a new understanding of what it means to "do math."

Although the title of this book is *My Best Idea: Math*, this article is about the collaborative work undertaken by Community Elders and leaders, non-Indigenous educators, and Algonquin and non-Indigenous students who came together to explore the connections between the mathematical content knowledge outlined in the mathematics curriculum and the mathematics inherent in Indigenous cultural practices (Beatty & Blair, 2015). The impetus for this work has been a national recognition of the need to explicitly incorporate Indigenous content to support identity building and an appreciation of Indigenous perspectives and values. In "Mathematics Exploration Through Algonquin Beadwork," we'll share a small part of the journey we have taken at a public school near Pembroke, Ontario. The school population comprises approximately 25% Algonquin students from the nearby Algonquins of Pikwakanagan First Nation and 75% non-Indigenous students. In this work, we brought together community and school in support of mathematics learning. We hope that the experiences presented are taken as an example of the possibilities of grounding math instruction in First Nations activities. Our goal is not to have teachers exactly recreate the instruction outlined here, but rather it is to establish relationships with local First Nations Community members to explore mathematics. Christina Ruddy, Operations Manager at the Pikwakanagan museum and cultural centre Omàmiwinini Pimàdjwowin (OP) and expert loomer, has articulated what we think is the most important aspect of this work — working with Indigenous Community members in the mathematics classroom:

> *I think it's important for a First Nations person to teach First Nations skills. If it doesn't have some kind of cultural significance, then what are we doing this for? Is it just to teach math? No. Not when you see the changes in the First Nations students. Seeing them more confident and the pride in talking with their peers about their lives, their regalia, and things like that. And it's nice to be able to share with your best friend who might not be Native a little bit more about your life that they might not know about because they only ever see you in a school setting.*

Culturally responsive mathematics education refers to efforts to make mathematics education more meaningful by aligning instruction with the cultural paradigms and lived experiences of students (Castagno & Brayboy, 2008). Our aim was to jointly explore the mathematics within the activity of bead looming to determine what mathematics made sense to the students — in effect, what math did students discover?

Consulting With the Community

The most important part of this process was to hold an initial consultation with Community members, who shared their insights and hopes for the children of the Community. During the course of the project, we made sure we met with these advisers regularly for guidance and feedback. During these discussions, Community members spoke about their ongoing work to revitalize Algonquin culture. As one Community member stated, "From the outside, it's perceived that we have all the culture. We don't. Culture was taken from us. We are actually in the process of learning our culture again." Like most Indigenous communities in Canada, the Algonquins of Pikwakanagan were forced to abandon many cultural practices, which they are now currently in the process of reviving. They saw this project as an opportunity to bring cultural revitalization into the classroom through activities such as beading, which has deep cultural and community significance. They also spoke about the importance of engendering pride in identity and saw the project as a way to support students to bring together Algonquin and other cultural perspectives.

Another key message was the recognition of the importance mathematics plays in being able to work within and alongside Canadian society. Community members articulated a desire to provide students with the opportunity to develop positive relationships with math. Seeing themselves in the math classroom gives students a chance to explore mathematical thinking in a way that reflects their culture. Community members also identified the difficulty many students have learning math, particularly those who struggle with pedagogical approaches that prioritize memorization. They believed that grounding math instruction in a context such as beading would give math more meaning and relevance.

Co-planning With Community Members

Over the course of this five-year project, we worked with a number of Community members to co-plan units of mathematics instruction based on traditional beadwork. This work extended from Grade 1 to Grade 8, but in this article, we will focus on the loom work done with students in Grade 3. All the projects supported the development of number sense, multiplicative thinking, algebraic reasoning, proportional reasoning, and spatial reasoning. During the co-planning phase, Christina taught the rest of the research team how to loom and shared her insights about some of the inherent mathematics of looming:

> *When you're deciding the width of the loom work, you count your beads, how long and how wide you want it. So, how long determines the number of columns and how wide determines the number of rows. There's a lot of counting. When you take your design and put it on the loom, then you need one extra thread on the loom for every count of beads that you do. For example, a bracelet that is 9 beads wide will need 10 warp threads on the loom. Then you have to measure your wrist and decide how long your bracelet needs to be.*

In addition to designing the lessons, we also asked Albert Owl, a fluent speaker of the Algonquin language, to teach the Algonquin language during the lessons. The Algonquins of Pikwakanagan First Nation have been actively working to revitalize the Algonquin language in the Community, and incorporating language instruction as part of culturally based math instruction meant that students could learn and use Anishinaabemowin in a meaningful context.

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Co-teaching in the Classroom

Prior to beginning our work in each of the grades, Christina shared with the students some of the history of beading and the importance of beadwork in Anishinaabe culture. She emphasized that looming is a traditional Algonquin activity that predates the arrival of Europeans in North America:

> *Before European people came here, this kind of work was done with all sorts of things you find in nature — shells, porcupine quills, and we used to trade for copper with communities living up along the north shore of Lake Superior. And then after that, people started trading for glass beads. I use glass and plastic beads in my work. We also used sinew from animals. Sinew is the muscles from an animal's legs that you dry, and it makes thread. Today we use mainly artificial sinew, leather cord, and thread.*

During co-teaching, Christina taught the art of beading to the students (she either taught alone or with another Community member). The classroom teacher and Christina then worked together to explore the mathematical concepts within the work, for example, identifying the rows and columns of the template or finding the unit of repeat, or the pattern core, for different patterns. We wanted to introduce students to an Algonquin activity taught by a Community member and to explore the mathematical ideas inherent in the activity. It is possible to engage in beadwork without explicitly thinking about the mathematics involved; however, we wanted to pay attention to and formalize the informal mathematical thinking that arose.

Grade 3 Explorations

Looming is a type of beading done on a loom, and it involves stringing beads onto vertical weft threads and weaving them through horizontal warp threads. Based on Christina's work, we created a design template of 20 columns and 5 rows. The columns represent the weft threads; on these threads, beads are threaded and woven onto the warp threads. The number of columns corresponds to the horizontal length of the beadwork. The rows represent the spaces between the warp threads, where the beads sit when they are woven into the work. The number of rows corresponds to the width of the beadwork. The rows and columns of the design space are numbered. Below each column, the number of each colour of bead is entered (with the number representing the colour of the beads, in this case, blue numbers for blue beads and red numbers for red beads). This helps the beader to know the order for stringing beads for each column, or line, of beads on the weft thread. Each column should add to the total number of beads on each weft thread (in Figure 1 below, each column should add up to 5).

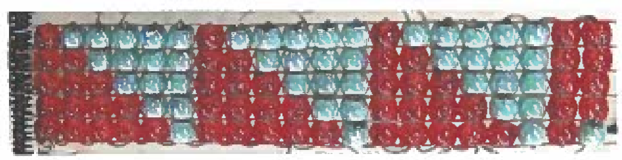

Figure 1

The initial patterns that students were taught were based on the geometric motifs that Christina used in her own looming. Christina modelled creating the staircase pattern (Figure 1) on a template that was 20 columns long and 5 rows wide. She filled in each successive column: column 1 with five blue squares, column 2 with four blue squares and one red square, column 3 with three blue squares and two red squares, and so on. As she filled in the columns, Anne George, the classroom teacher, built the pattern by creating columns of Unifix® cubes. Christina filled in the template to the 13th column and then asked the students to copy and extend the pattern onto their own 20-column templates.

Copying the pattern onto the paper template allowed the students to get a sense of the space they would be working within. Once students had designed their patterns using the paper template, they then used manipulatives to further explore their patterns. Through the process of filling in the paper template and then completing the concrete models, the students began to notice characteristics of the pattern. For example, many students quickly realized that the staircase pattern was a repeating one and that the first six columns represented the part of the pattern that repeats. Students seemed to orient to the column structure of the pattern in order to identify the unit of repeat, which may have been because that was how Christina and Anne had introduced the pattern.

The students were then introduced to more complicated patterns. The flower pattern (Figure 2) easily discernible, and students identified the unit of repeat, which they called the pattern core, as comprising 5 columns.

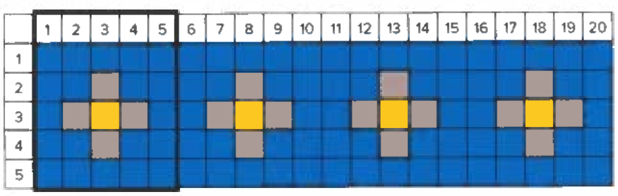

Figure 2

Anne: If we have a core that is 5 columns long, then how many times does it repeat on our 20-column template?

Ryan: There's 20 columns on the template, and 5 plus 5 plus 5 plus 5 is 20.

Anne wrote 5 + 5 + 5 + 5 = 20 on the board.

Figure 3, which the students named green X, posed some difficulty.

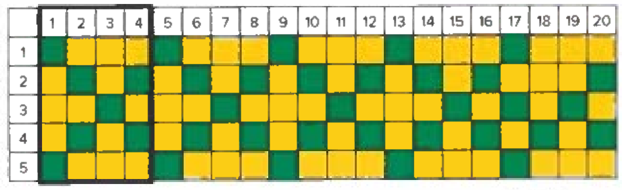

Figure 3

Jayna: There's something odd about it.

Anne: What's odd about it?

Jayna: Because the second pattern core attaches to the first core.

Anne: Does column 1 repeat anywhere?

Ellie: Yes, in column 5. And then in 9 ... 13 ... 17.

Anne drew arrows to illustrate what Ellie described: that every fourth column was identical.

Anne: Where does the pattern start repeating again?

Students: At 5.

Anne: So, we know that a pattern is something that repeats. What is repeating?

Luis: Columns 1 to 4, then 5 to 8 are the same.

Sam: And now 9 to ... 13 ...

Borden: 9 to 12.

Jayna: Then 13 to ...

Sam: 16. And 16 to 20.

Luis: Wait ... 17 to 20.

Anne: So, this is our last one here (*circling the final four columns on the template*).

Luis: Then it would be 21.

Anne: 21 to what?

Ellie: 21 to 25! Because it's plus 4. It's counting by 4s.

The students initially thought that the unit of repeat was made up of 5 columns, likely because the previous pattern had units of repeat that were 5 columns. They might also have thought this because the green X extended from columns 1 to 5. However, prompted by Anne's question, further investigation showed that the first column repeated in column 5, and so the unit of repeat was made up of columns 1 to 4 and began again at column 5. They then mapped out the units of repeat along the 20 columns (Figure 4) and were able to make predictions that extended beyond the 20-column template.

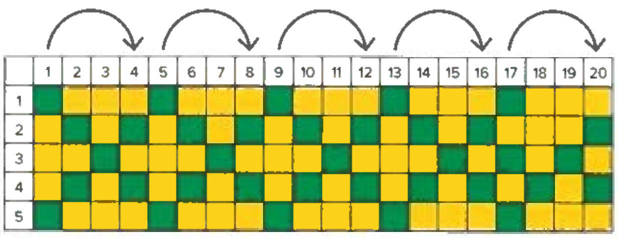

Figure 4

Anne asked the students how many times they had counted by 4 columns from column 1 to column 20.

Anne: Let's look at our pattern. How many columns were in that pattern core, and how many times does that column repeat?

Students: 5!

Ruth: How come?

Jade: 4, 8, 12, 16, 20!

Anne wrote 4 + 4 + 4 + 4 + 4.

The students discovered that for the flower pattern, four iterations of the 5-column unit of repeat filled the 20 columns, while on the green X pattern, five iterations of the 4-column unit of repeat filled the 20 columns. Therefore, in one lesson, students were developing an idea of multiplicative thinking and the commutative property of multiplication.

Christina asked the students how many beads they would need for each template. The students calculated that for the green X pattern, there were 20 beads per unit of repeat because it was 4 groups (columns) of 5 beads (rows) and that the group of 20 repeated 5 times (units of repeat). Using skip counting by 20s, they determined that there were 100 beads in the template. They then looked at the flower pattern to determine that there were 25 beads per core and that this group of 25 repeated 4 times, which also gave 100 beads.

The last pattern (Figure 5) introduced to the students was a chevron pattern. In order to make identifying the unit of repeat easier, Anne put up special math tools called occluders on the board. These could be dragged to hide different parts of the pattern in order to help students discern which columns were repeating. This lent itself to an interesting discussion because some students attended to the first orange chevron and saw the unit of repeat from columns 3 to 6. Others saw the unit of repeat as columns 1 to 4, which contained a full green chevron.

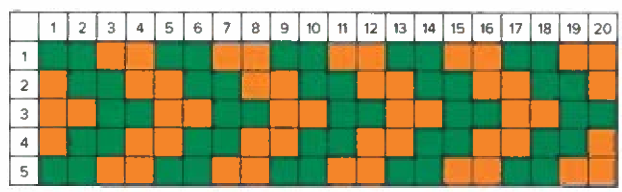

Figure 5

At first, there was speculation that there might be two units of repeat, from columns 1 to 4 and from columns 3 to 6. Then students wondered if the entire unit of repeat comprised columns 1 to 6, but they realized this could not be the case since column 1 and column 7 were different; therefore, column 7 was not the beginning of a new unit of repeat. Through class discussion, students determined the pattern core was made up of columns 1 to 4. Anne moved the occluders to highlight the succession of cores (1–4, 5–8, etc.), and students agreed that each was an iteration of the original unit of repeat. She then took a sample of the unit of repeat and pulled it over the completed template to demonstrate how the unit repeated along the template.

She asked the students how many times the core repeated.

> Anne: How many of the core did I find in my 20 columns?
>
> Luis: Oh, now it makes sense ... Five times 4!
>
> John: Five times of 4.
>
> Anne: Let's write that thinking down. [*Writes 5 × 4.*] Five groups of 4 ...
>
> Students: Equals 20.

Through these investigations, students co-constructed an understanding of multiplicative thinking, and the language of multiplication was introduced. When determining the unit of repeat, students identified groups of columns that made up the unit, or pattern core, and then focused on how many iterations of the unit was possible on the 20-column template. They then progressed from skip counting by 5s or 4s to repeated addition to thinking about "groups of." They also engaged in an initial exploration of the commutative property of multiplication. In addition, students also explored the number of beads in each unit and used that to determine the total number of beads in one template. When considering the number of beads, instead of the number of columns, students skip counted by units of 20 or 25 to determine the total number of beads.

During the next discussions, students were asked to make predictions farther down the sequence of the pattern based on their understanding of the unit of repeat. For example, Roman and John analyzed the following pattern (Figure 6).

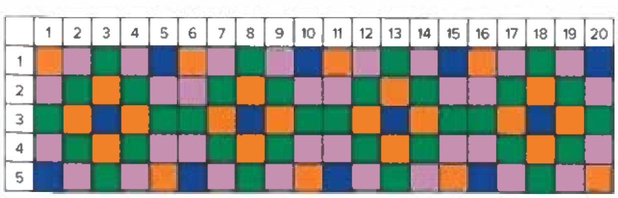

Figure 6

They identified the unit of repeat as 5 columns by noticing that columns 1 and 6 were the same; therefore, column 6 represented the beginning of a new unit of repeat, or pattern core. When asked what the 40th column would look like, they reasoned it would look like the 20th column, and also the 5th, 10th, 15th, and 25th column because, as one student said, "It's the last column of the core. And if there were 40 columns, the core would repeat 8 times because 4 times 5 is 20, so 8 times 5 is 40." When asked to predict the 39th column, students reasoned it would look like the 4th column because it would be the column before the last column of the pattern core. They made a final generalization, stating that by looking at the pattern, they realized that any column ending in a 5 or 0 would be identical to the 5th column, any column ending in 4 or 9 would look like the 4th column, any column ending in 3 or 8 would look like the 3rd column, any column ending in 2 or 7 would look like the 2nd, and any column ending in 1 or 6 would look like the 1st. This is a recognition of the co-variation between the units of repeat made up of numbered columns and the elements in the unit of repeat (the bead patterns of each column).

Looming

Christina taught the students how to string a loom and how to transfer their two-dimensional patterns to three-dimensional beadwork. The students discovered that the order of the individual squares down each column on the template represented the order with which beads were strung on the weft threads and that these were then woven onto the warp threads. The act of creating the beadwork by stringing and weaving columns of beads provided students with an opportunity to internalize their patterns. This physical enactment, picking up and stringing sequences of beads, meant that the students could internalize both the complex repetitions of the pattern (sequences of beads in each column and sequences of columns in each unit of repeat, or pattern core) and experience the physical growth of the pattern. This allowed for the development of deep conceptual understanding of the composition of the core of a pattern, the repetition of that core, and the relationships among multiple iterations of that core as the pattern grew.

Although the initial patterns introduced were simple, the students created their own unique complex designs (Figure 7).

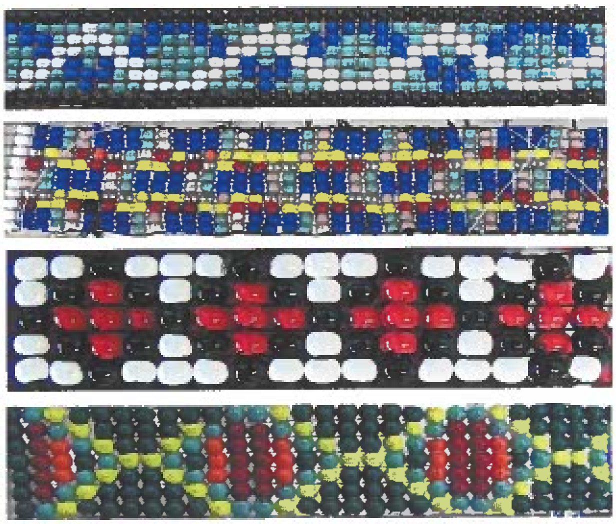

Figure 7

Mathematical Thinking: Summary

The activities in this study supported the development of many kinds of mathematical thinking, including multiplicative thinking, algebraic reasoning, and proportional reasoning.

One of the most important concepts underpinning multiplicative thinking is unitizing (Lamon, 1996). Unitizing is the ability to perceive and consider a unit simultaneously as one unit and as the collection of elements creating the unit. Through the process of creating repeating patterns on the template and then building them with linking cubes, students created units of repeat (which they referred to as pattern cores). The design template is set up as an array of rows and columns, and students determined the unit of repeat in their patterns by identifying the columns that made up the unit and identifying the column where the unit seemed to begin again. This became easy for patterns like the diagonal and the flower pattern (the unit of repeat was columns 1 to 5), for which students discerned that the unit of repeat comprised 5 columns.

Other patterns, such as the green X, were more complex because they had a unit of repeat made up of 4 columns. This was initially more difficult to determine because the green X embedded in the design of the pattern made it appear at first that the unit of repeat was also made up of 5 columns. However, when the students looked from the perspective of repeating columns, they noticed that columns 1 and 6 were not identical but that columns 1 and 5 were identical, and so the unit of repeat comprised 4 columns. Drawing rectangles to delineate all the units, they noticed the 4-column core repeated 5 times across the 20-column template and, comparing it to the previous pattern, made the discovery that a 4-column unit would repeat 5 times, but a 5-column unit would repeat 4 times. This brought them to a discussion of the commutativity of multiplication — that 5 groups of 4 columns (or, as one student stated, 5 times of 4) equalled 20 columns, and 4 groups of 5 columns also equalled 20 columns. One student explained that for the 4-column units, he imagined that one column was taken off each of the 5-column units, which would result in one more 4-column unit that could fit on the template.

Students also used the array structure of the template to calculate the total number of beads in each unit of repeat and in each template. For example, in a 5-column unit of repeat, since the template had 5 rows, students recognized that each unit of repeat was made up of 25 individual beads. They then skip counted by 25s to find the total number of beads. Students also skip counted by units of 20 (for patterns with a 4-column unit of repeat), units of 5 (the number of beads in each column), and units of 10 (the number of beads in 2 columns).

Once students were able to determine the units of repeat, they extended this understanding to be able to make next, near, and far predictions. Most elementary patterning curricula focus on asking students to determine what comes next in a repeating pattern. In our study, the focus of the analysis went beyond identifying the unit of repeat and predicting what comes next to being able to use the structure of the pattern to make accurate predictions farther down the sequence. Physically creating bracelets provided an opportunity for students to have to think beyond the 20 columns of the given template. Looming meant that the repeating patterns were physically growing, and students reported internalizing the repetition of the pattern in terms of sequences of columns of beads, which led to an ability to formulate generalizations. For example, students could use the structure of the 5-column core of a pattern, coupled with the numbered columns, to be able to predict that any column ending in a 5 or 0 would be identical to the 5th column, any column ending in 9 or 4 would look like the 4th column, and so on.

This recognition of the co-variation between the numbered columns and the elements in the unit of repeat (the beads in each column) was extended to other kinds of patterning activities. One of these activities came from the mandatory provincial math assessment that Grade 3 students complete every year (Figure 8).

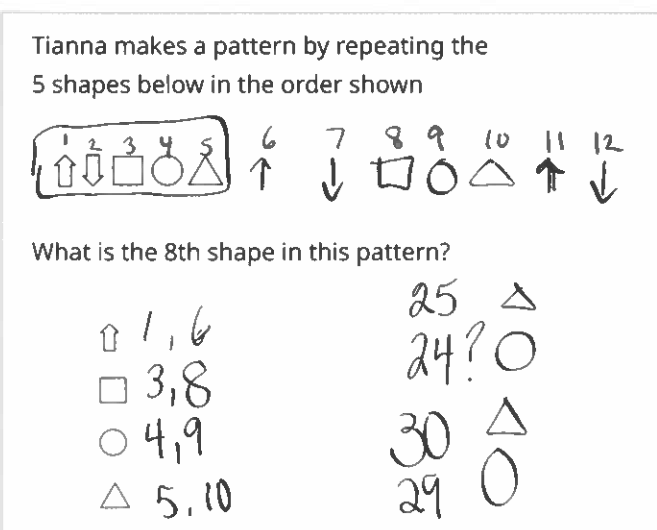

Figure 8

In this activity, students are asked to predict what the 8th shape in a repeating sequence of 5 shapes will be. The item can be answered by simply extending the pattern and counting to the 8th shape. However, students in our study realized they were analyzing the same kind of structure as their looming patterns and were able to articulate a generalization predicting the correct shapes anywhere in the sequence. For example, they reasoned that any multiple of 5 subtract 1 would be a circle and that the 108th shape would be a square because the 110th shape (a multiple of 5) would be a triangle, and then subtract (go back) 2 shapes.

Another critical and related math concept is the ability to reason proportionally (Lamon, 1996). In the Grade 6 classroom, students used the proportion of 5 columns = 1 cm to determine how long their bracelets needed to be. In this task, the template represented 20 columns, 4 cm, and 1 core simultaneously. Once they knew how many centimetres they needed, the students calculated the total number of columns required by multiplying the total number of centimetres by 5 columns. They then divided the total number of columns by the number of templates (or cores) needed to find out how many full templates they would need and how many columns would be left over (which would need to be incorporated into the finished design). The mathematical proportional calculations provided the foundation for the finished design.

These activities also support students' spatial reasoning skills, which have been linked to mathematical achievement (Mathewson, 1999; Gunderson et al., 2012). For all students, the process of creating and analyzing two-dimensional patterns on a grid provided an opportunity to engage in visuospatial thinking. The planning and beading activities ensured students considered many different spatial relationships, including beads within a column, the relationship of the columns that made up the pattern core, and the composition of the bracelet as a whole. In all activities, students considered both numeric quantities and their spatial representations, for example, the number of beads and columns in a core and the spatial composition of the design.

Algonquin Language in Math Class

Along with mathematics exploration, a part of every lesson was devoted to teaching students words and phrases in Algonquin that were related to the activity of beading. As Mr. Owl taught students words, he also explained some of the meaning behind the words. The word for bead, *manidominens*, comes from the word *Manido*, Creator or spirit, and *minens*, meaning small piece, so a bead is a small piece of the Creator, or a small spirit. Students learned about the importance of respecting the materials they were using and the importance of putting good thoughts into their work.

Mr. Owl further explained the Algonquin language so that students learned, for example, the suffix *àtig* means that an object is made out of wood. The words for loom, *mazinàbido-iganàtig*, and pencil, *ojibihiganàtig*, both end with this suffix. Both words refer to something made out of wood that helps you to do something — a loom is used to create a picture with beads and a pencil is used to make marks on paper. Students also learned words for colours so that they could describe their patterns in Algonquin. Introducing the Algonquin language through the activity of looming helped students develop a meaningful connection to the Language (Figure 9).

Figure 9

Integrating a traditional cultural practice as the basis for mathematical instruction led to increased levels of enjoyment and engagement for all students as well as high levels of mathematical thinking.

Positive Math Experiences

Integrating a traditional cultural practice as the basis for mathematical instruction led to increased levels of enjoyment and engagement for all students as well as high levels of mathematical thinking. Although the lessons supported students grappling with complex concepts such as algebraic and proportional reasoning, the means of facilitating this learning was through the process of designing and constructing beaded bracelets. Students spent up to three hours at a time in the classroom not only working on designs but also solving mathematical problems that emerged from the activities.

Conclusion

This project was designed to investigate the potential of situating math instruction within Algonquin cultural activities. We have given only a few examples of the kinds of cultural connections and robust mathematical thinking that we have documented, but this project has had a profound influence on everyone involved and on the larger school context.

All students had an opportunity to learn from the Community members who came into the classroom to teach, and students could see that the knowledge brought by those members was honoured and respected. Christina taught the lessons with an emphasis on the cultural importance of the activity, which provided an opportunity for students from Pikwakanagan to connect to their own cultural heritage. It also led to a greater understanding of what it means to "do math," with the underlying message that everyone is a mathematician, and complex mathematical thinking is part of Algonquin culture. This realization was also important for the non-Indigenous students, who gained greater insights into the culture of their classmates, and they extended their own mathematical thinking. It is our hope that this project can serve as an example of how valuing First Nations knowledge systems and creating integrated instruction may respond to the Truth and Reconciliation Commission's calls to action, particularly 63.iii: "building student capacity for intercultural understanding, empathy, and mutual respect" (Truth and Reconciliation Commission of Canada: Calls to Action, p. 7).

Finally, in order to ensure that First Nations, Metis, and Inuit cultures are not appropriated in this kind of work, we would like to re-emphasize that the most important aspect is building relationships. Non-Indigenous teachers must work alongside (and sometimes behind) Community members when integrating FNMI perspectives in the mathematics classroom, allowing Community members to take the lead (P. Agawa, personal communication, January 2018).

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THE BOTTOM LINE

- Work in partnership with Community members to create culturally responsive education experiences.
- Align instruction with the cultural paradigms and lived experiences of students.
- Incorporate Indigenous content to support identity building and an appreciation of Indigenous perspectives and values.
- Recognize that complex mathematical thinking is part of Algonquin culture.
- Value First Nations knowledge systems. Create integrated instruction to help build student capacity for intercultural understanding, empathy, and mutual respect.

Ruth Beatty teaches the mathematics methods course for teacher candidates in the Faculty of Education at Lakehead University. At present, she is working with members of Anishinaabe communities and educators from northern Ontario school boards to explore how to incorporate Indigenous ways of knowing mathematics into the Ontario math curriculum.

Christina Ruddy, the Algonquins of Pikwakanagan First Nation, is Operations Manager at the Algonquins of Pikwakanagan First Nation museum and cultural centre Omàmiwinini Pimàdjwowin (OP) and an expert loomer.